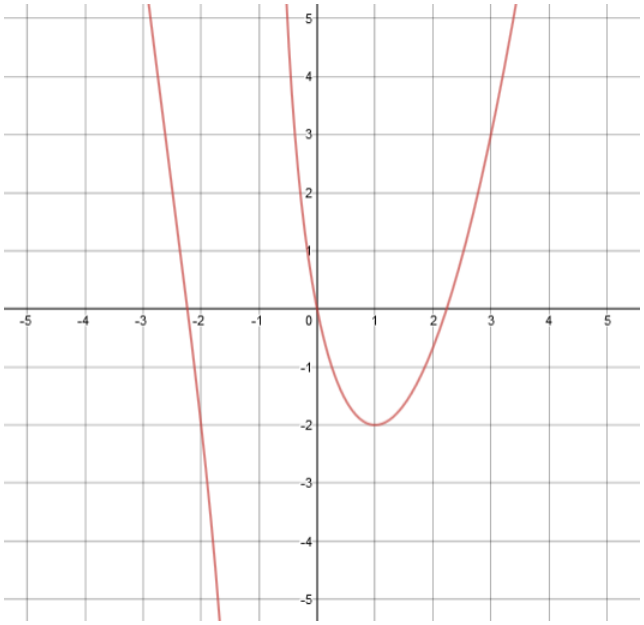
5.1 Graphs of Reciprocal Functions

A General Rules

Let consider a *continuous* function $y = f(x)$ and its *reciprocal* $g(x) = \frac{1}{f(x)}$. Here are some general rules:

1. If $y = f(x) > 0$ ($y = f(x) < 0$), then $g(x) = \frac{1}{f(x)} > 0$ ($g(x) = \frac{1}{f(x)} < 0$).
2. If the function $y = f(x)$ is *even/odd/neither*, then the reciprocal function $g(x) = \frac{1}{f(x)}$ is also *even/odd/neither*.
3. If the function $y = f(x)$ is *increasing/decreasing* over an interval, then the reciprocal function $g(x) = \frac{1}{f(x)}$ is *decreasing/increasing* over the same interval.
4. If the function $y = f(x)$ has a *local/global minimum/maximum* at $(a, f(a))$, the reciprocal function $g(x) = \frac{1}{f(x)}$ has a *local/global maximum/minimum* at $(a, g(a)) = (a, \frac{1}{f(a)})$.
5. If the function $y = f(x)$ has a *horizontal asymptote* $y = a$ ($y \to a$ as $x \to \pm\infty$), then the reciprocal function $g(x) = \frac{1}{f(x)}$ has *horizontal asymptote* $y = \frac{1}{a}$.
6. If the function $y = f(x)$ is *unbounded* ($y \to \pm\infty$) as x becomes *unbounded* ($x \to \pm\infty$), then the reciprocal function $g(x) = \frac{1}{f(x)}$ has the *horizontal asymptote* $y = 0$.
7. If the function $y = f(x)$ has a *real zero* at $x = a$, then the reciprocal function $g(x) = \frac{1}{f(x)}$ has a *vertical asymptote* $x = a$.
8. If the function $y = f(x)$ has a *vertical asymptote* $x = a$ ($y \to \pm\infty$ as $x \to a$), then the reciprocal function $g(x) = \frac{1}{f(x)} \to 0$ as $x \leftarrow a$.

Ex 1. The function $y = f(x)$ is represented graphically below. Use the general rules to graph its reciprocal function $g(x) = \frac{1}{f(x)}$.



Ex 2. Graph the linear function $f(x) = 2x - 4$ and its reciprocal $g(x) = \frac{1}{2x-4}$ on the same grid.

Ex 3. Graph the quadratic function $f(x)=x^2+2$ and its reciprocal $g(x)=\frac{1}{x^2+2}$ on the same grid.	Ex 4. Graph the quadratic function $f(x)=(x-1)^2$ and its reciprocal $g(x)=\frac{1}{(x-1)^2}$ on the same grid.
Ex 5. Graph the quadratic function $f(x)=(x-1)(x+2)$ and its reciprocal $g(x)=\frac{1}{(x-1)(x+2)}$ on the same grid.	Ex 6. Graph the cubic function $f(x)=x^3-1$ and its reciprocal $g(x)=\frac{1}{x^3-1}$ on the same grid.
Ex 7. Graph the function $f(x)=x^3-x^2-2x$ and its reciprocal $g(x)=\frac{1}{x^3-x^2-2x}$ on the same grid.	Ex 8. Graph the function $f(x)=\lvert x^2-4\rvert$ and its reciprocal $g(x)=\frac{1}{\lvert x^2-4\rvert}$ on the same grid.

Reading: Nelson Textbook, Pages 248-254
Homework: Nelson Textbook, Page 254: #1, 3, 5eg, 6, 8acdf, 9ac, 11, 15abc, 16